Signal Propagation in Nth LC Ladder Network Using Fibonacci Wave Functions

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Abstract—In this paper, new general model for an infinite LC ladder network using Fibonacci wave functions (FWF) is introduced. This general model is derived from a first order resistive-capacitive (RC) or resistive-inductive (RL) circuit. The n^{th} order Fibonacci wave function of an LC ladder denominator and numerator coefficients are determined from Pascal's triangle new general form. The coefficients follow specific distribution pattern with respect to the golden ratio. The LC ladder network model can be developed to any order for each inductor current or flux and for each capacitor voltage or charge. Based on this new proposed method, n^{th} order FWF general models were created and their signal propagation behaviors were compared with n^{th} order RC and LC electrical circuits modeled with Matlab-Simulink. These models can be used to represent and analyze lossless transmission lines and other applications such as particles interaction behavior in quantum mechanics, sound propagation model.

Index Terms— Fibonacci wave functions, LC ladder, Pascal's triangle, Golden ratio, Transmissions lines.

I. INTRODUCTION

Fibonacci wave functions (FWFs) are transfer functions with high degree that are irreducible. Their characteristic behavior is unique. Their response to a step input signal gives multiple intermediate stationary regimes before reaching the final steady state which presents oscillations with low amplitudes. The FWFs can be created theoretically from a first-order origin wave function [1]. Fibonacci wave functions have multiple resonance and anti-resonance frequencies organized in a perfect way with respect to each other. Moreover, they have two well defined Fibonacci boundary systems using Pascal's triangle [2].

In this paper, a step by step development methodology of new electrical circuit application of FWFs called Fibonacci electrical circuits (FECs) is introduced to model perfectly the recurrent LC ladder network. These FECs can be used to model transmission cables [3], [4], the behavior and interaction of the infinitely small particles using the infinite LC networks [5] in quantum mechanics, the neural dynamic in biology [6], etc.

The paper is organized as follow. Section II describes a general model of resistive-capacitive Fibonacci electrical circuit (RC-FEC). Section III presents a comparative study of Fibonacci wave functions (FWFs) model and its equivalent Matlab-Simulink RC-FEC circuit. Section IV describes a general model of resistive-inductive Fibonacci electrical circuit (RL-FEC). Section V gives a comparative study of Fibonacci wave functions (FWFs) and its equivalent Matlab-Simulink RL-FEC circuit. Nth order RC-FEC and RL-FEC FWF general models are presented in section VI. Section VII presents an application of FWF to transmission lines and are compared with particular case of short circuit found in [3].

II. RC FIBONACCI ELECTRICAL CIRCUIT (RC-FEC)

The original Fibonacci wave function has the following form.

$$g_1^{(k,x_c)}(s) = \frac{K}{s + x_c} \quad (1)$$

With

$$K = \omega_f^2 \quad \text{and} \quad x_c = 2 \xi_{cf} \omega_f$$

The first order electrical circuit resistive-capacitive Fibonacci electrical circuit (RC-FEC) is presented in figure 1.

Fig 1. First order RC-FEC

$$\begin{aligned} \frac{V_o}{I_i} &= \frac{1}{s + \frac{1}{RC}} = L \frac{1}{s + \frac{1}{RC}} = L \frac{K}{s + x_c} = L g_1^{(K,x_c)}(s) \\ \left(\frac{V_o}{I_i}\right)_1^{(K,x_c)} &= \frac{K}{s + x_c} = g_1^{(K,x_c)}(s) \\ K &= \frac{1}{LC} \quad \text{and} \quad x_c = 2 \xi_{cf} \omega_f = \frac{1}{RC} \end{aligned} \quad (2)$$

The second order RC-FEC circuit diagram is shown in figure 2 and its wave function in (3).

Fig 2. Second order RC-FEC

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$$V_i = (L g_1^{(K,x_c)}(s) + sL) I_o = \frac{LC (s + g_1^{(K,x_c)}(s)) I_o}{C}$$

$$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)} = \frac{K}{s + \frac{K}{s + x_c}} = g_2^{(K,x_c)}(s) \quad (3)$$

The wave function of the third order RC-FEC (4) is derived from circuit diagram presented in figure 3

Fig 3. Third order RC-FEC

$$I_i = (C g_2^{(K,x_c)}(s) + sC) V_o = \frac{LC (s + g_2^{(K,x_c)}(s)) V_o}{L}$$

$$\left(\frac{V_o}{LI_i}\right)^{(K,x_c)} = \frac{K}{s + \frac{K}{s + \frac{K}{s + x_c}}} = g_3^{(K,x_c)}(s) \quad (4)$$

One can see that an even n^{th} order RC-FEC (figure 4) will have voltage as input and current as output.

$$n = n_c + n_L \quad (5)$$

n_c is the total number of capacitors in the circuit.
 n_L is the total number of inductance in the circuit.

Fig 4. n^{th} even order RC-FEC

The FWF of this circuit is.

$$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)} = \frac{K}{s + g_{n-1}^{(K,x_c)}(s)} = g_n^{(K,x_c)}(s) \quad (6)$$

For n^{th} odd order, the wave function is.

$$\left(\frac{V_o}{LI_i}\right)^{(K,x_c)} = \frac{K}{s + g_{n-1}^{(K,x_c)}(s)} = g_n^{(K,x_c)}(s) \quad (7)$$

Fig 5. n^{th} odd order RC-FEC

In general, n^{th} even order RC-FEC with voltage as input and current as output has a final steady-state value Cx_c .
For n^{th} odd order RC-FEC with current as input and voltage as output have a final steady-state value $L \frac{K}{x_c}$.

Table I. RC-FEC Fibonacci wave functions

$\left(\frac{V_o}{LI_i}\right)^{(K,x_c)}_1 = g_1^{(K,x_c)}$ $= \frac{K}{s + x_c}$	$\left(\frac{V_o}{LI_i}\right)^{(K,x_c)}_1 = g_1^{(K,x_c)} = \frac{K}{1s + 1x_c}$
$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)}_2 = g_2^{(K,x_c)}$ $= \frac{K}{s + \frac{K}{s + x_c}}$	$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)}_2 = g_2^{(K,x_c)}$ $= \frac{Ks + Kx_c}{1s^2 + 1x_cs + 1K}$
$\left(\frac{V_o}{LI_i}\right)^{(K,x_c)}_3 = g_3^{(K,x_c)}$ $= \frac{K}{s + \frac{K}{s + \frac{K}{s + x_c}}}$	$\left(\frac{V_o}{LI_i}\right)^{(K,x_c)}_3 = g_3^{(K,x_c)}$ $= \frac{Ks^2 + Ksx_c + K^2}{1s^3 + 1x_cs^2 + 2Ks + 1Kx_c}$
$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)}_4 = g_4^{(K,x_c)}$ $= \frac{K}{s + \frac{K}{s + \frac{K}{s + \frac{K}{s + x_c}}}}$	$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)}_4 = g_4^{(K,x_c)}$ $= \frac{Ks^3 + Kx_cs^2 + 2K^2s + K^2x_c}{1s^4 + 1x_cs^3 + 3Ks^2 + 2Kx_cs + 1K^2}$
$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)}_n = g_n^{(K,x_c)}$ $= \frac{K}{s + g_{n-1}^{(K,x_c)}(s)}$ <i>n even</i>	$g_n^{(K,x_c)}(s) = \frac{Kden_{n-1}^{(K,x_c)}(s)}{sden_{n-1}^{(K,x_c)}(s) + num_{n-1}^{(K,x_c)}(s)}$
$\left(\frac{V_o}{LI_i}\right)^{(K,x_c)}_n = g_n^{(K,x_c)}$ $= \frac{K}{s + g_{n-1}^{(K,x_c)}(s)}$ <i>n odd</i>	$g_n^{(K,x_c)}(s) = \frac{Kden_{n-1}^{(K,x_c)}(s)}{den_n^{(K,x_c)}(s)}$

III. SIMULATION OF RC-FEC AND FWFs

Simulation studies were conducted to compare the previous electrical circuits with Fibonacci wave functions and Matlab-Simulink electrical circuit model. The studies confirm that these circuits follow the logic of a recurrent Fibonacci sequence and can be modelled by FWFs.

A. Case 1: $R=1\Omega$; $L=1H$; $C=1F$

In this case $(K, x_c) = (1,1)$. Pascal's Triangle in table II will be used to determine all FWFs.

Order 14 FWF taken as example is an even function, using its numerator and denominator coefficients are expressed in (8) using table II.

$$\left(\frac{I_o}{CV_i}\right)^{(K,x_c)}_{14} = g_{14}^{(K,x_c)}(s) = \frac{Kden_{13}^{(K,x_c)}(s)}{den_{14}^{(K,x_c)}(s)}$$

$$den_{13}^{(K,x_c)}(s) = 1s^{13} + 1x_cs^{12} + 12Ks^{11} + 11Kx_cs^{10} + 55K^2s^9 + 45K^2x_cs^8 + 120K^3s^7 + 84K^3x_cs^6 + 126K^4s^5 + 70K^4x_cs^4 + 56K^5s^3 + 21K^5x_cs^2 + 7K^6s + 1K^6x_c \quad (8)$$

$$den_{14}^{(K,x_c)}(s) = 1s^{14} + 1x_cs^{13} + 13Ks^{12} + 12Kx_cs^{11} + 66K^2s^{10} + 55K^2x_cs^9 + 165K^3s^8 + 120K^3x_cs^7 + 210K^4s^6 + 126K^4x_cs^5 + 126K^5s^4 + 56K^5x_cs^3 + 28K^6s^2 + 7K^6x_cs + 1K^7$$

Table II. Pascal's triangle general form with multiplication coefficients.

For comparison purpose, simulations of $(\frac{I_o}{V_i})_{40}^{(1,1)} = C * g_{40}^{(1,1)}(s)$ FWF model and Matlab-Simulink RC-FEC electrical circuit model order 40 are illustrated in figure 6. The final steady-state is $C * \frac{k}{x_c} = 1$ with unit input voltage.

Fig 6. FWF RC-FEC $(\frac{I_o}{V_i})_{40}^{(1,1)} = C * g_{40}^{(1,1)}(s)$ and Matlab-Simulink model with $V_i = 1V$

The $(\frac{V_o}{I_i})_{39}^{(K,x_c)} = L * g_{39}^{(K,x_c)}(s)$, which is odd order, will be determined using Pascal's triangle table II.

Simulation results of FWF $(\frac{V_o}{I_i})_{39}^{(1,1)} = L * g_{39}^{(1,1)}(s)$ and Matlab-Simulink RC-FEC electrical circuit order 39 model are compared and are identical as illustrated in figure 7. The final steady-state is $L * x_c = 1$ with unit input current.

Fig 7. FWF RC-FEC $(\frac{V_o}{I_i})_{39}^{(1,1)} = L * g_{39}^{(1,1)}(s)$ and Matlab-Simulink model with $I_i = 1A$

IV. RL FIBONACCI ELECTRICAL CIRCUIT (RL-FEC)

RL-FEC is determined in the same way as RC-FEC. The first order circuit in figure 8 and its FWF is presented in (9).

Fig 8. First order RL-FEC

$$\frac{I_o}{V_i} = \frac{1}{s + \frac{R}{L}} = C \frac{1}{s + \frac{R}{L}} = C \frac{K}{s + x_L} = C g_1^{(K,x_L)}(s)$$

$$(\frac{I_o}{CV_i})_1^{(K,x_L)} = \frac{K}{s + x_L} = g_1^{(K,x_L)}(s) \quad (9)$$

$$K = \frac{1}{LC} \quad \text{and} \quad x_L = 2 \xi_{Lf} \omega_f = \frac{R}{L}$$

$$K = \frac{1}{LC} = x_c x_L = 4 \xi_{cf} \xi_{Lf} \omega_f^2$$

$$\xi_{cf} \xi_{Lf} = \frac{1}{4}$$

The 2nd order RL-FEC will be defined with current input and voltage output (figure 9) and its FWF expressed in (10).

Fig 9. Second order RL-FEC

$$I_i = (C g_1^{(K,x_L)}(s) + sC) V_o$$

$$= \frac{LC (s + g_1^{(K,x_L)}(s)) V_o}{L} \quad (10)$$

$$(\frac{V_o}{LI_i})_2^{(K,x_L)} = \frac{K}{s + \frac{K}{s + x_L}} = g_2^{(K,x_L)}(s)$$

The third order RL-FEC will be defined with an input voltage and output current (Figure 10) and its FWF in (11).

Fig 10. First order RL-FEC

$$V_i = (L g_2^{(K,x_L)}(s) + sL) I_o$$

$$= \frac{LC (s + g_2^{(K,x_L)}(s)) I_o}{C}$$

$$\left(\frac{I_o}{CV_i}\right)_3^{(K,x_L)} = \frac{K}{s + \frac{K}{s + x_L}} = g_3^{(K,x_L)}(s)$$

In general, RL-FEC with an even n^{th} order in figure 11 has current as input and voltage as output.

$$n = n_c + n_L$$

n_c is the total number of capacitors in the circuit.
 n_L is the total number of inductance in the circuit.

Fig 11. n^{th} even order RL-FEC

The wave function is (13).

$$\left(\frac{V_o}{LI_i}\right)_n^{(K,x_L)} = \frac{K}{s + g_{n-1}^{(K,x_L)}(s)} = g_n^{(K,x_L)}(s)$$

A RL-FEC with n^{th} odd order (Figure 12) has voltage as input and current as output and its wave function expressed in (14).

$$\left(\frac{I_o}{CV_i}\right)_n^{(K,x_L)} = \frac{K}{s + g_{n-1}^{(K,x_L)}(s)} = g_n^{(K,x_L)}(s)$$

In general, n^{th} even order RL-FEC has a current input and voltage output with a final steady-state value of $L * x_L$.
 n^{th} odd order RL-FEC with voltage as input and current as output has a final steady-state value of $C * \frac{K}{x_L}$.

Fig 12. n^{th} odd order RL-FEC

Table III. RL-FEC Fibonacci wave functions

$\left(\frac{I_o}{LV_i}\right)_1^{(K,x_L)} = g_1^{(K,x_L)}$ $= \frac{K}{s + x_L}$	$\left(\frac{I_o}{LV_i}\right)_1^{(K,x_L)} = g_1^{(K,x_L)} = \frac{K}{1s + 1x_L}$
$\left(\frac{V_o}{LI_i}\right)_2^{(K,x_L)} = g_2^{(K,x_L)}$ $= \frac{K}{s + \frac{K}{s + x_L}}$	$\left(\frac{V_o}{LI_i}\right)_2^{(K,x_L)} = g_2^{(K,x_L)} = \frac{Ks + Kx_L}{1s^2 + 1x_Ls + 1K}$
$\left(\frac{I_o}{LV_i}\right)_3^{(K,x_L)} = g_3^{(K,x_L)}$ $= \frac{K}{s + \frac{K}{s + \frac{K}{s + x_L}}}$	$\left(\frac{I_o}{LV_i}\right)_3^{(K,x_L)} = g_3^{(K,x_L)}$ $= \frac{Ks^2 + Ksx_L + K^2}{1s^3 + 1x_Ls^2 + 2Ks + 1Kx_L}$
$\left(\frac{V_o}{LI_i}\right)_4^{(K,x_L)} = g_4^{(K,x_L)}$ $= \frac{K}{s + \frac{K}{s + \frac{K}{s + \frac{K}{s + x_L}}}}$	$\left(\frac{V_o}{LI_i}\right)_4^{(K,x_L)} = g_4^{(K,x_L)}$ $= \frac{Ks^3 + Kx_Ls^2 + 2K^2s + K^2x_L}{1s^4 + 1x_Ls^3 + 3Ks^2 + 2Kx_Ls + 1K^2}$
$\left(\frac{V_o}{LI_i}\right)_n^{(K,x_L)} = g_n^{(K,x_L)}$ $= \frac{K}{s + g_{n-1}^{(K,x_L)}(s)}$ <i>n even</i>	$g_n^{(K,x_L)}(s) = \frac{Kden_{n-1}^{(K,x_L)}(s)}{sden_{n-1}^{(K,x_L)}(s) + num_{n-1}^{(K,x_L)}(s)}$
$\left(\frac{I_o}{LV_i}\right)_n^{(K,x_L)} = g_n^{(K,x_L)}$ $= \frac{K}{s + g_{n-1}^{(K,x_L)}(s)}$ <i>n odd</i>	$g_n^{(K,x_L)}(s) = \frac{Kden_{n-1}^{(K,x_L)}(s)}{den_n^{(K,x_L)}(s)}$

V. SIMULATION OF RL-FEC AND FWFs

The simulations will be made with the same values as RC-FEC.

A. Case 1: $R=1\Omega$; $L=1H$; $C=1F$; $(K, x_L) = (1,1)$.

$$x_L = \frac{R}{L} = 1; \quad K = \frac{1}{LC} = 1$$

Simulations results of the FWF $\left(\frac{V_o}{I_i}\right)_{40}^{(1,1)} = Lg_{40}^{(1,1)}(s)$ and Matlab-Simulink RL-FEC electrical circuit model order 40 are shown in the figure 13 below. The final and the only steady-state is:

$$\frac{K}{x_L} = 1 = x_c = x_L$$

Fig 13. FWF RL-FEC $\left(\frac{V_o}{I_i}\right)_{40}^{(1,1)} = Lg_{40}^{(1,1)}(s)$ and Matlab-Simulink model with $I_i = 1A$

The wave function $\left(\frac{I_o}{V_i}\right)_{39}^{(1,1)} = Cg_{39}^{(1,1)}(s)$, is shown in figure 14 with its equivalent Matlab-Simulink electrical circuit RL-FEC model order 39, and both are exactly identical.

Fig 14. FWF RL-FEC $\left(\frac{I_o}{V_i}\right)_{39}^{(1,1)} = Cg_{39}^{(1,1)}(s)$ and Matlab-Simulink model with $V_i = 1V$

An n^{th} order RL-FEC and its FWF behaves exactly the same way as an n^{th} order RC-FEC and its FWF only if $x_L = x_c$.

$$\left(\frac{V_o}{L I_i}\right)_n^{(K, x_L)} = g_n^{(K, x_L)} = \left(\frac{I_o}{C V_i}\right)_n^{(K, x_c)} = g_n^{(K, x_c)} \text{ with } x_L = x_c$$

$$x_L = x_c = \frac{R}{L} = \frac{1}{RC} \quad (15)$$

$$R = \sqrt{\frac{L}{C}}$$

$$K = \omega_f^2; x_L = x_c = \omega_f = \frac{1}{\sqrt{LC}}$$

Fig 15. RL-FEC and RC-FEC order 40 ; $\left(\frac{V_o}{I_i}\right)_{40}^{(1,1)} = Lg_{40}^{(1,1)}(s)$, $I_i = 1A$)
 $\left(\left(\frac{I_o}{V_i}\right)_{40}^{(1,1)} = C * g_{40}^{(1,1)}(s), V_i = 1V\right)$

VI. N^{th} ORDER LC LADDER RC-FEC AND RL-FEC GENERAL MODEL

The n^{th} order RC-FEC and RL-FEC are modeled. All currents through each inductor and voltages through each capacitor are perfectly determined by Fibonacci wave functions illustrated in section II. The model is illustrated in figure 17 for $n = 40$ and can be extended to an infinite order knowing that each FWF can be determined using Pascal's triangle general form in table II.

Fig 16. n^{th} even order RC-FEC

Fig 17. n^{th} even order RC-FEC Model using FWFs for each current and voltage branch.

This model can be presented with the variable charge $Q_j^{(K, x_c)} = C V_j^{(K, x_c)}$ in each capacitor and the electromagnetic flux $\phi_j^{(K, x_c)} = L I_j^{(K, x_c)}$ in each inductor as shown in figure 18.

Fig 18. n^{th} even order RC-FEC Model using FWFs for each flux and charge branch.

In the same way, the RL-FEC model is illustrated in figure 19 below for $n = 40$.

Fig 19. n^{th} even order RL-FEC

Fig 20. n^{th} even order RL-FEC Model using FWFs for each current and voltage branch.

This model can be also presented with the variable charge $Q_j^{(K,x_L)} = CV_j^{(K,x_L)}$ in each capacitor and the electromagnetic flux $\phi_j^{(K,x_L)} = LI_j^{(K,x_L)}$ in each inductor as shown in figure 21.

Fig 21. n^{th} even order RL-FEC Model using FWFs for each flux and charge branch.

The simulation was conducted for RC-FEC using Matlab-Simulink electrical circuit and the corresponding FWF model for $n = 40$ to confirm the accuracy of the proposed model for all currents and voltages. Note that this model is applicable to any order n of the RC-FEC and any order n of RL-FEC.

The figures 22 and 23 show simulation for both RC-FEC models for voltage V_{37} and current I_{40} for the case $(K, x_c) = (1, 4)$.

A. Case 2: $K=1$, $x_c = 4$, $R=10\Omega$.

$$K = \frac{1}{LC} = \omega_f^2 = x_L x_c$$

$$C = \frac{1}{R * x_c} = 0.025F \quad (16)$$

$$L = \frac{1}{K * C} = 40H$$

$$x_L = \frac{R}{L} = 0.25$$

Fig 22. Matlab-Simulink RC-FEC circuit and proposed FWF general model for $I_{40}^{(1,4)}$ with input $V_i = 1V$

Fig 23. Matlab-Simulink RC-FEC circuit and proposed FWF general model for $V_{37}^{(1,4)}$ with input $V_i = 1V$

FWFs model presented in figures 17 and 20 are exactly the same as Matlab-Simulink RC-FEC and RL-FEC circuits of all inductors currents and all capacitors voltages. Thus the RC-FEC and RL-FEC can be shaped for any order due to the fact that every order is well defined with its Fibonacci wave function precisely determined from Pascal's triangle.

Figures 24 and 25 show the behavior of all voltages for each capacitor (odd FWFs) and all currents in each inductor (even FWFs). Simulation results show also the delay of each branch based on its position from the input source.

Fig 24. Matlab-Simulink RC-FEC and FWF general model ($V_1^{(1,4)}$, $V_3^{(1,4)}$, ..., $V_{39}^{(1,4)}$)

Fig 25. Matlab-Simulink RC-FEC and FWF general model ($I_2^{(1,4)}$, $I_4^{(1,4)}$, ..., $I_{40}^{(1,4)}$)

For the case where both RC-FEC and RL-FEC have the same time constant $x_c = x_L$. Simulation was conducted to confirm the accuracy of the FWF general model proposed in figure 17 and 20 and Matlab-Simulink circuit model.

B. Case 1: $R=1\Omega$; $L=1H$; $C=1F$

$$\begin{aligned} C &= \frac{1}{R * x_c} = 1F \\ L &= \frac{1}{K * C} = 1H \\ x_L &= \frac{R}{L} = x_c = \frac{1}{R * C} = 1 \\ R &= \sqrt{\frac{L}{C}} = 1 \\ K &= \frac{1}{LC} = \omega_f^2 = x_L x_c = x_L^2 = x_c^2 \end{aligned} \quad (17)$$

The simulation in figure 26 and 27 for Matlab-Simulink RC-FEC and its FWF model for order 40 shows no intermediate steady states and this due to the fact that

$$\begin{aligned} K &= \frac{1}{LC} = \omega_f^2 = x_L x_c = x_L^2 = x_c^2 \\ x_c &= 2\xi_{cf}\omega_f = x_L = 2\xi_{Lf}\omega_f \end{aligned}$$

Fig 26. Matlab-Simulink RC-FEC and FWF general model ($I_{40}^{(1,1)}$, $V_i = 1V$, $R = \sqrt{\frac{L}{C}}$)

Fig 27. Matlab-Simulink RC-FEC and FWF general model ($V_{11}^{(1,1)}$, $V_i = 1V$, $R = \sqrt{\frac{L}{C}}$)

The figures 28 and 29 show the behavior of all voltages in each capacitor (odd order) and all currents of each inductor (even order) using FWF general model of figure 17 and compared with Matlab-Simulink RC-FEC circuit (figure 16).

Fig 28. Matlab-Simulink RC-FEC and FWF general model ($V_1^{(1,1)}$, $V_3^{(1,1)}, \dots, V_{39}^{(1,1)}$, $R = \sqrt{\frac{L}{C}}$)

Fig 29. Matlab-Simulink RC-FEC and FWF general model ($I_2^{(1,1)}, I_4^{(1,1)}, \dots, I_{40}^{(1,1)}$, $R = \sqrt{\frac{L}{C}}$)

RC-FEC and RL-FEC general model can be also derived based on the energy for each capacitor and inductor.

For RC-FEC energy general model for $n = 40$ is presented.

$$\begin{aligned} (e_c)^{(K,x_c)}_{2j+1} &= \frac{1}{2} C (V_{2j+1}^{(K,x_c)})^2 \\ (e_L)^{(K,x_c)}_{2j} &= \frac{1}{2} L (I_{2j}^{(K,x_c)})^2 \\ \frac{(e_L)^{(K,x_c)}_{2j}}{(e_c)^{(K,x_c)}_{2j+1}} &= \frac{(g_{2j}^{(K,x_c)}(s))^2}{\omega_f^2} \\ \frac{(e_c)^{(K,x_c)}_{2j+1}}{(e_L)^{(K,x_c)}_{2j}} &= \frac{(g_{2j+1}^{(K,x_c)}(s))^2}{\omega_f^2} \end{aligned} \quad (18)$$

Below is the energy general model for RC-FEC $n = 40$ with input energy e_i^m .

Fig 30. n^{th} even order RC-FEC general model using energy wave between branches.

VII. FIBONACCI WAVE FUNCTIONS APPLIED TO TRANSMISSION LINES

In the literature, many papers have studied and modeled the transmission lines [3] with different analytical methods but very few have noticed that Fibonacci numbers and especially Pascal's triangle can be used [3], [4].

It is well known that the communications lines can be modeled with a recurrent LC depending on the length of the cable. There have been studies to analyze the input impedance as well as the load impedance to better understand the reflection phenomena when the input impedance is

different from the load. In [3], it was shown that in a transmission cable with a short-circuit ($R=0\Omega$), the input impedance or admittance can be found using Pascal's triangle for the case $L=1H$ and $C=1F$. This is a particular case of Pascal's triangle general form detailed in table II with $K=1$ and $x_c = \infty$.

Below is an example of $g_{14}(s)$ using Pascal's triangle above.

$$\begin{aligned} den_{14}^{(k,x)} &= 1s^{14} + 1xs^{13} + 13Ks^{12} + 12Kxs^{11} + 66K^2s^{10} \\ &\quad + 55K^2xs^9 + 165K^3s^8 + 120K^3xs^7 \\ &\quad + 210K^4s^6 + 126K^4xs^5 + 126K^5s^4 \\ &\quad + 56K^5xs^3 + 28K^6s^2 + 7K^6xs \\ &\quad + 1K^7 \\ den_{13}^{(k,x)} &= 1s^{13} + 1xs^{12} + 12Ks^{11} + 11Kxs^{10} + 55K^2s^9 \\ &\quad + 45K^2xs^8 + 120K^3s^7 + 84K^3xs^6 \\ &\quad + 126K^4s^5 + 70K^4xs^4 + 56K^5s^3 \\ &\quad + 21K^5xs^2 + 7K^6s + 1K^6x \\ den_{12}^{(k,x)} &= 1s^{12} + 1xs^{11} + 11Ks^{10} + 10Kxs^9 + 45K^2s^8 \\ &\quad + 36K^2xs^7 + 84K^3s^6 + 56K^3xs^5 \\ &\quad + 70K^4s^4 + 35K^4xs^3 + 21K^5s^2 \\ &\quad + 6K^5xs + 1K^6 \end{aligned} \quad (19)$$

For RL-FEC circuit in figure 19 for order 13 and 14

$$\begin{aligned} \left(\frac{I_o}{CV_i}\right)_{13}^{(K,x_L)} &= g_{13}^{(K,x_L)}(s) = \frac{Kden_{12}^{(K,x_L)}(s)}{den_{13}^{(K,x_L)}(s)} \\ \left(\frac{V_o}{LI_i}\right)_{14}^{(K,x_L)} &= g_{14}^{(K,x_L)}(s) = \frac{Kden_{13}^{(K,x_L)}(s)}{den_{14}^{(K,x_L)}(s)} \\ x &= x_L = \frac{R}{L} \end{aligned} \quad (20)$$

For RC-FEC circuit in figure 16 for order 13 and 14.

$$\begin{aligned} \left(\frac{I_o}{CV_i}\right)_{14}^{(K,x_c)} &= g_{14}^{(K,x_c)}(s) = \frac{Kden_{13}^{(K,x_c)}(s)}{den_{14}^{(K,x_c)}(s)} \\ \left(\frac{V_o}{LI_i}\right)_{13}^{(K,x_L)} &= g_{13}^{(K,x_c)}(s) = \frac{Kden_{12}^{(K,x_c)}(s)}{den_{13}^{(K,x_c)}(s)} \\ x &= x_c = \frac{1}{RC} \end{aligned} \quad (21)$$

Note that based on which Fibonacci circuit, either RC-FEC or RL-FEC, one can easily determine the input impedance or input admittance for any order n and for both short-circuit and open-circuit using only general Pascal's triangle as shown in (19), (20) and (21).

For the purpose of comparison with [2] for a short-circuit case ($R = 0\Omega$), $x_c = \infty$ using RC-FEC and $x_L = 0$ using RL-FEC. Equations (19), (20) and (21) become.

$$\begin{aligned} \left(\frac{V_o}{LI_i}\right)_{14L}^{(K,0)} &= g_{14L}^{(K,0)}(s) = K * \frac{1s^{13} + 12Ks^{11} + 55K^2s^9 + 120K^3s^7}{1s^{14} + 13Ks^{12} + 66K^2s^{10} + 165K^3s^8} \\ &\quad + 210K^4s^6 + 126K^5s^4 + 28K^6s^2 + 1K^7 \\ \left(\frac{I_o}{CV_i}\right)_{13L}^{(K,0)} &= g_{13L}^{(K,0)}(s) = K * \frac{1s^{12} + 11Ks^{10} + 45K^2s^8 + 84K^3s^6}{1s^{13} + 12Ks^{11} + 55K^2s^9 + 120K^3s^7} \\ &\quad + 126K^4s^5 + 56K^5s^3 + 7K^6s \\ \left(\frac{I_o}{CV_i}\right)_{14c}^{(K,\infty)} &= g_{14c}^{(K,\infty)}(s) = K * \frac{1s^{12} + 11Ks^{10} + 45K^2s^8 + 84K^3s^6}{1s^{13} + 12Ks^{11} + 55K^2s^9 + 120K^3s^7} \\ &\quad + 126K^4s^5 + 56K^5s^3 + 7K^6s \end{aligned} \quad (22)$$

$$\left(\frac{V_o}{LI_i}\right)_{13c}^{(K,\infty)} = g_{13c}^{(K,\infty)}(s) = K * \frac{1s^{11} + 10Ks^9 + 36K^2s^7}{1s^{12} + 11Ks^{10} + 45K^2s^8 + 84K^3s^6} + 70K^4s^4 + 21K^5s^2 + 1K^6$$

For an open-circuit ($R = \infty\Omega$), $x_c = 0$ using RC-FEC and for RL-FEC $x_L = \infty$. Equations (19), (20) and (21) become.

$$\begin{aligned} \left(\frac{I_o}{CV_i}\right)_{14c}^{(K,0)} &= g_{14c}^{(K,0)}(s) = K * \frac{1s^{13} + 12Ks^{11} + 55K^2s^9 + 120K^3s^7}{1s^{14} + 13Ks^{12} + 66K^2s^{10} + 165K^3s^8} \\ &\quad + 210K^4s^6 + 126K^5s^4 + 28K^6s^2 + 1K^7 \\ \left(\frac{V_o}{LI_i}\right)_{13c}^{(K,0)} &= g_{13c}^{(K,0)}(s) = K * \frac{1s^{12} + 11Ks^{10} + 45K^2s^8 + 84K^3s^6}{1s^{13} + 12Ks^{11} + 55K^2s^9 + 120K^3s^7} \\ &\quad + 126K^4s^5 + 56K^5s^3 + 7K^6s \\ \left(\frac{V_o}{LI_i}\right)_{14L}^{(K,\infty)} &= g_{14L}^{(K,\infty)}(s) = K * \frac{1s^{12} + 11Ks^{10} + 45K^2s^8 + 84K^3s^6}{1s^{13} + 12Ks^{11} + 55K^2s^9 + 120K^3s^7} \\ &\quad + 126K^4s^5 + 56K^5s^3 + 7K^6s \\ \left(\frac{I_o}{CV_i}\right)_{13L}^{(K,\infty)} &= g_{13L}^{(K,\infty)}(s) = K * \frac{1s^{11} + 10Ks^9 + 36K^2s^7}{1s^{12} + 11Ks^{10} + 45K^2s^8 + 84K^3s^6} \\ &\quad + 70K^4s^4 + 21K^5s^2 + 1K^6 \end{aligned} \quad (23)$$

Simulation for short and open circuit were conducted using FWF general model and Matlab-Simulink RC-FEC and RL-FEC for $n = 40$. $V_{40}^{(1,x_L=0)}$, $I_{21}^{(1,x_L=0)}$, $I_{40}^{(1,x_c=0)}$, $V_{21}^{(1,x_c=0)}$ are shown in figure 31, 32, 33 and 34.

With open-circuit ($R = \infty\Omega$) or short-circuit ($R = 0\Omega$) RL-FEC and RC-FEC and their respective FWF general model show continuous oscillations for constant unit input and for pulse unit input and are exactly same as Matlab-Simulink circuit model.

Fig 31. Matlab-Simulink RL-FEC and FWF general model ($V_{40}^{(1,0)}$, $I_i = 1A$, $R = 0\Omega$)

Fig 32. Matlab-Simulink RL-FEC and FWF general model ($I_{21}^{(1,0)}$, $I_i = 1A$, $R = 0\Omega$)

Fig 33 Matlab-Simulink RC-FEC and FWF general model ($V_{21}^{(1,0)}$, $V_i = 1V$ 35s pulse, $R = \infty\Omega$)

Fig 34. Matlab-Simulink RC-FEC and FWF general model ($I_{40}^{(1,0)}$, $V_i = 1V$ 35s pulse, $R = \infty\Omega$)

VIII. CONCLUSION

In this paper, a complete LC ladder general model based on Fibonacci wave functions FWFs that was introduced. The importance for LC ladder comes from its application that can be found in the literature like lossless transmission lines model, the sound propagation model in the ear and in quantum mechanics to understand the interaction between the particles. The detailed model that is proposed for each LC ladder branch with precise Fibonacci wave functions shows that the FWF general model and its corresponding Matlab-Simulink circuit_model are perfectly same for each inductor current and capacitor voltage with defined load or in short ($R = 0\Omega$) or open load ($R = \infty\Omega$). The FWF general model using the charge in the capacitor and the flux in the inductor for each branch is also presented for all charge and flux LC ladder branches.

The LC ladder input impedance or admittance can be derived using Pascal's triangle general form presented in section VII with defined coefficients K , x_L and x_C .

Transmission lines, short-circuit and open-circuit were studied and simulated for both Fibonacci model and Fibonacci electrical circuit to show that these cases are particular cases of general model and their Fibonacci wave functions are easy to determine based on Pascal's triangle for short load ($x_C = \infty$ and $x_L = 0$) and for open load ($x_L = \infty$ and $x_C = 0$).

The LC ladder general model for energy transfer between L and C in each section can be used for many other applications that use lossless LC recurrent circuit as model for more research and analysis especially, in quantum mechanics, biology and communication.

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