Design and Implementation of MIPS Processor for LU Decomposition based on FPGA

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Abstract—The solution for a set of linear equations require to find the matrix inverse of a square matrix with same number of the linear equations, this operation require many mathematical calculations. To solve this problem, LU decomposition for the matrix is used, which computes two matrices, a lower triangle matrix and an upper triangle matrix. In this, paper a design for 32-bits MIPS (microprocessor without interlocked pipelined stages) processor with the required instructions that used to calculate the LU matrices. The design implemented using VHDL (Very high speed integrated circuit hardware description language) then integrated with FPGA (Field Programmable Gate Arrays) Xilinx Spartan 6. The results for the different parts of the processor are resented in the form of test bench waveform and the architecture of the system is demonstrated and the results was matched with theoretical results.

Index terms—Field Programmable Gate Array (FPGA), LU decomposition, VHDL, MIPS Processor.

I. INTRODUCTION

Matrix inversion is a well-known operation that be used by several algorithms, for example in solving simultaneous equations. The matrix inversion requires a lot of mathematical operations that cannot able to perform the operation in real time demand. An alternative method is by using the LU decomposition instead of the matrix inversion.

The LU-decomposition method is first “decomposes” matrix A into two matrices L and U. More accurately, if A is an $n \times n$ matrix, the first matrix is Lower triangular and the second matrix is an Upper triangular, both were an $n \times n$ matrices as shown below in the following set equations for a 3×3 matrix:

$$A = LU \quad (1)$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} \quad (2)$$

$$\text{Where, } U_{11} = a_{11}, U_{12} = a_{12}, U_{13} = a_{13} \quad (3)$$

$$L_{i1} = \frac{a_{i1}}{U_{11}}, L_{21} = \frac{a_{21}}{U_{11}}, L_{31} = \frac{a_{31}}{U_{11}}, \quad (4)$$

$$U_{22} = a_{22} - L_{21} \times U_{12}, U_{23} = a_{23} - L_{21} \times U_{13} \quad (5)$$

and

$$a_{32} = U_{12} \times L_{31} + U_{22} \times L_{32} \quad (6)$$

$$a_{33} = L_{31} \times U_{13} + L_{32} \times U_{23} + U_{33} \quad (7)$$

For system of the equations $AX=B$, the motivation for LU decomposition is based on the observation of equations involving triangular matrices are easier to deal with .

When obtaining the LU decomposition of matrix A, then solving the system of n linear equations in n variables very effectively in the following two substitutions:

$$AX=B \leftrightarrow LUX=B, \quad (8)$$

$$UX=Y \text{ (by many back substitutions)} \quad (9)$$

$$LY=B \text{ (by many forward substitutions)}$$

The goal of this project is to build a 32-bit MIPS processor that be able to perform the LU decomposition. The processor is designed and implemented using VHDL.

Following sections are organized as follows: section two gives an explanation for the MIPS processor and the design procedure. Section 3 shows how the data were represented in this work; following sections give the results in details and the conclusions respectively.

II. MIPS PROCESSOR

Single cycle MIPS processor is a RISC (Reduced Instruction Set Computer) which can perform an entire instruction in one cycle. slowest instruction can limit cycle time. The MIPS processor contains 32-bits general purpose registers. In this paper a single cycle MIPS processor is designed and implemented to perform the mathematical requirements to apply the LU decomposition.

A. MIPS Instruction Set Architecture (ISA)

Microprocessors have two types of instruction set, first Reduced Instruction Set Computer (RISC) and second Complex Instruction Set Computer (CISC). RISC based processors have reduced number of instructions, large set of general purpose registers, instructions of fixed length and simplified address modes.

RISC processors works on the principle of load-store architecture. Whereas CISC based processors have more number of instructions with different varieties of address modes and variable length instructions.

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RISC processors when compared to CISC processors will consume less area to implement execute faster and have better performance making RISC processors suitable for system on chip circuitry [3] [4].

MIPS architecture contains 32-bits general-purpose registers where the operands are either registers or memory locations as shown in table 1, and instructions of length 32 bits. The processor, for word access it uses byte addressing.

TABLE 1: MIPS general purpose registers

Name	Register Number	Usage	Preserved on call
\$zero	0	Constant value 0	n.a
\$at	1	Reserved for the assembler	n.a
\$v0-\$v1	02-Mar	Value for results and expressions	no
\$a0-\$a3	04-Jul	Arguments (procedures/factions)	yes
\$t0-\$t7	Aug-15	Temporaries	no
\$s0-\$s7	16-23	Saved	yes
\$t8-\$t9	24-25	More temporaries	no
\$k0-\$k1	26-27	Reserved for the operating system	n.a
\$gp	28	Global pointer	yes
\$sp	29	Stack pointer	yes
\$fp	30	frame pointer	yes
\$ra	31	Return pointer	yes

B. Instruction Formats

MIPS have 3 instruction format, namely: R-format, I-format and J-format. Table 2 shows these instruction formats.

All instructions that do not have a memory address as an operand, target offset and an immediate value comes under R-format instructions. Operations are performed on the data in the registers.

A total of 3 registers are required for the completion of an operation, two registers Rs and Rt as operands and the third register Rd to store the result of the operation. All arithmetic and shift instructions come under this format. Op-code '000000' is used by all the instructions, which are under this format [5].

Branch instructions, load and store instructions and all the instructions that have an immediate operand comes under I-format instructions. Op-codes such as '000000', '0100xx' and '00001x' are not used by I-format instructions [6].

Jump instructions J and JAL come under J-format instructions. The operands of these instructions are specified by memory address. Op-code '00001x' is used to specify J-format instructions [5].

TABLE 2: MIPS Instruction format

Field size	6-bits	5-bits	5-bits	5-bits	5-bits	6-bits
R-Type	op	Rs	Rt	Rd	shamt	funct
I-Type	op	Rs	Rt	16-bits Imm		
J-Type	op	26-bits Addr				

Where,

- op: Operation code (also known as opcode) that selects a certain operation.
- rs: First source register.
- rt: Second source register.
- rd: Destination register.
- func: Is used together with the opcode to select an arithmetic instruction.
- address: the 26-bits address which is used in jump instruction.
- Shamt: used for shift instruction to hold the shift amount
- Imm: 16 bits address is used within data transfer and control instructions.

C. Single cycle MIPS Processor Design

A single-cycle MIPS is a type of MIPS microarchitecture.

MIPS executes every instruction in one clock cycle [7], where:

1) Slowest instruction can limit cycle timing which make shorter instructions execute in one of the unnecessary long cycle [8].

2) Additionally, data-path resource can't be used more than once per each instruction, else must be duplicated [9]. And because of it the single cycle design will need:

- Instruction and data as two memories.
- And two adders, one that computes value of (PC+4) while the other computes branch target address.

The MIPS implementation consists of two different types of logic elements:

1) Combinational: an elements that operate on data values such as ALU and adder.

2) Sequential: an elements that have and hold a state like register file, instruction memory and data memory [8] [10].

For design simplicity, an edge-triggered clocking methodology is assumed that allow state element to read and write at the same clock cycle. Fig.1 shows two state elements placed round combinational logic, that operate in single clock cycle.

signals must transit from state element one, through combinational logic to the state element two in one clock cycle. The time is necessary for signals to reach state element two that defines the length of the clock cycle [11].

Figure. 1. State elements, ccombinational logic, and clock cycle are closely related [11].

The MIPS microarchitecture is divided into two interacting parts:

1) Datapath: 32-bits datapath implicate elements such as registers, memories, sign-extenders, multiplexers, datapath elements as showing below:

a) Program counter (PC): it is a 32bit register and its output represents the address of the current instruction (instr) while input automatically incremented by 4 (depends on instr size 4 bytes) to read PC next represent instruction address.

b) Instruction memory take 32bit address from a) Program counter register and read 32bit data in output port.

c) Register file combine of 32 registers each. Having two reading ports (RD1) and (RD2) and one writing port (WD3). Reading ports (RD1) and (RD2) take 5bit address inputs (A1) and (A2) which select one of the registers to be read on the reading output ports. Writing port will take 5bit address input (A3) which select one register to which the 32bit on (WD3) input port will be written if the (WE) signal is one on the raising of the clock signal.

d) Data memory have one output reading port (RD) and one input writing port (WR). 32-bit data at input (WR) port is written to the memory location which can be specified by address (A) if (WE) signal is one at the raising of the clock signal.

e) Adder such an Arithmetic Unit (AU) with the label (+) to indicate that it is always performs an add operation on its inputs.

f) Multiplexer (Mux) used to chose only one input from multi inputs and pass that to output. Mux select one line that controlled by a signal which provided by control unit.

g) Sign extender simply copy the sign bit (most significant bit) of half word or byte to all upper word output.

h) AU (Arithmetic Unit) designed to execute all of the arithmetic instructions. arithmetic unit (AU) consists of addition and subtraction (add/sub) which all versions of FPGA kits supported for these operations, multiplication (mul) which some types of FPGA kits support like (Virtex 4, Virtex 7, spartan 3E and spartan 6), also Spartan 6 device and any top series family can support division (div).

2) Control unit (CU): receives the “opcode” “instr 31:26” and function “instr5:0” fields of current instruction which is received from instruction memory. The design consists of eight signals (8 control signals), (alu control) signal used to for arithmetic operation as shown in table 4. Designing set of instructions as shown in table 3 and figure for internal architecture, which is suitable to perform LU decomposition.

TABLE 3: Instruction set

Instruct ions	SW RD,R	LW RD,R	AD D	ADDi RD,R	SUB RD,R	MUL RT, RS, RD	DIV RT, RS, RD
	S,IM M	S,IM M	RD, RS, RT	S,IM M	RS,R T	RS, RD	RS, RD

Opcod e	1010 11	1000 11	0	1000	0	0	0
Regwri te	0	1	1	1	1	1	1
Regdst	0	0	1	0	1	1	1
ALUS RC	1	1	0	1	0	0	0
ZERO	0	0	0	0	0	0	0
MEM Write	1	0	0	0	0	0	0
MEMt oRegist er	0	1	0	0	0	0	0
ALUop code	0	0	10	0	10	10	10
Functio n	x	x	1000 00	x	1000 10	10010 0	1001 01
type	I- Type	I- Type	R- Type	I- Type	R- Type	R- Type	R- Type
Descri ption	Save word in mem ory	Load word from mem ory	Addi tion oper ation	Addi tion imme diate opera tion	Subtr action opera tion	Multi plicati on operat ion	Divi sion operat ion

TABLE 4: Function of (alu control) signal

Code	Operation
0	mul
1	div
10	add
11	not used
100	not used
101	not used
110	sub
111	not used

Figure. 2. Control unit internal architecture

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Figure. 3. MIPS processor for LU decomposition

III. DATA REPRESENTATION

There are two types of data representation (fixed point and floating point).

The arithmetic of fixed point is an extension of integer representation that allows relatively reducing the range of numbers with a constant precision as well as implementation at low complexity.

The use of arithmetic of the floating point is much expensive in regarding hardware and it leads to inefficient designs especially to implement FPGA.

The word length of the fixed point that has been used is equal to 32bits divided into 20bits fraction point, 11 bits integer part and last bit been sign number. Figure 4 shows the design of fixed point format.

Fixed point arithmetic results in efficient hardware designs. Our design uses two's complement fixed point arithmetic, The data lines used in implementation for fixed point arithmetic consist of an integer part, a fractional part and a sign bit.

Sign	Integer value	decimal fraction
1 bit	11 bits	20 bits
31	30 20	19 0

Figure. 4. Fixed point format

IV. SIMULATION RESULTS

The designed processor was implemented using the Xilinx ISE design suite 14.7 tool. Figure 5 shows the RTL schematic top level and figure 6 shows the RTL schematic for architecture of the MIPS processor.

To test the designed processor, a matrix has been implemented using VHDL and Xilinx Spartan 6 by using a

set of instructions that calculated the LU decomposition, The A matrix and its LU decomposition is shown in equation (11) for 3x3 matrix which also processor can perform up to 8x8 matrix.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} = L \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix} U \begin{bmatrix} 1 & 2 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \quad (11)$$

Figure. 5. RTL top level

Figure 7 shows the test bench for the simulation waveform for the matrix A and its LU decomposition and in figure 8 shows the resource required for implemented design.

Figure. 6. RTL schematic for the MIPS processor

Figure. 7. Test Bench results

Device Utilization Summary (estimated values)			
Logic Utilization	Used	Available	Utilization
Number of Slice Registers	7	54576	0%
Number of Slice LUTs	5014	27288	18%
Number of fully used LUT-FF pairs	0	5021	0%
Number of bonded IOBs	67	296	22%
Number of BUFG/BUFGCTRLs	1	16	6%
Number of DSP48A1s	4	58	6%

Figure. 8. Summarizes the FPGA resources

V. CONCLUSIONS

In this paper, 32-bit MIPS processor was designed with certain instructions that are sufficient to perform the LU decomposition operation by using VHDL Xilinx ISE software language and FPGA kit Spartan 6. The implemented design tested and gives correct results at running frequency 50 MHz

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