

Three-Phase Distribution System Load Flow Analysis Using Sequence Components

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Abstract — Electric power distribution systems are usually very unbalance. Therefore, a power flow method that can handle the three-phase configuration of the power system is needed so that the system planning and operation can properly be carried out. In the case of three-phase distribution system power flow analysis, for each system bus (except for substation bus), the voltage magnitude and angle of the three phases must be calculated. These calculations are carried out under certain loading conditions. After these voltages have been calculated, the electric power flows and losses in the distribution lines, and the substation power can also be determined. This paper proposes a new technique for three-phase distribution system power flow analysis using sequence components. The new formulation for the power flow problem in terms of sequence components is also proposed and developed in this paper. The application of sequence components has the advantage that the size of the problem can effectively be reduced, and solution to the power flow problem will be easier to find. Case study using a representative distribution test system confirms the validity of the proposed method where comparative studies between the proposed (i.e. sequence components based) method and the phase components based method are carried out.

Index Terms — Electric distribution network, unbalance system, load flow analysis, sequence components.

I. INTRODUCTION

Power flow (or load flow) is an important tool in operational study and planning of electric power systems. The single-phase power flow algorithm assumes that the unbalance in an electric power system is small and can be neglected so that the system can be considered to be balanced. There are many cases where system unbalance can no longer be ignored. This can occur for example due to: (i) load unbalance, (ii) transmission/distribution network unbalance, and (iii) combination of (i) and (ii). Thus, a three-phase power flow method that can be used on an unbalanced system is needed so that the system can be analyzed.

Several researchers have developed some interesting three-phase power flow algorithms to analyze the unbalanced distribution systems. These algorithms can be grouped into two categories: (i) algorithms that use the Newton-Raphson iteration scheme [1]-[5]; and (ii) algorithms that utilize the radial structure of the power distribution system (forward/backward sweeping techniques) [6]-[11].

Unbalanced power systems can be modeled using phase

components. However, the application of sequence components has the advantage that the size of the problem can effectively be reduced. For power systems with n buses, this size will be reduced from $3n \times 3n$ admittance matrix (i.e. phase admittance matrix) to three $n \times n$ admittance matrices (i.e. zero, positive and negative sequence admittance matrices). With this reduction, the complexity of the power flow problem formulation will also be reduced, and therefore, it will be easier to solve.

The present work investigates the three-phase distribution system load flow (DSLFL) method using sequence components. The new formulation for the power flow problem in terms of sequence components is proposed and developed in this paper. Comparative studies between the proposed method and the phase components based method are also carried out. These comparative studies are conducted to validate the proposed power flow method using sequence components.

II. THEORY OF SEQUENCE COMPONENTS

A. Voltage and Current

By using the symmetrical component theory, phase voltage and current can be expressed in terms of sequence voltage and current as follows [12]:

$$\mathbf{V}^{abc} = \mathbf{A}\mathbf{V}^{012} \quad (1)$$

$$\mathbf{I}^{abc} = \mathbf{A}\mathbf{I}^{012} \quad (2)$$

where:

$$\mathbf{V}^{abc} = \begin{bmatrix} V^a \\ V^b \\ V^c \end{bmatrix}; \mathbf{V}^{012} = \begin{bmatrix} V^0 \\ V^1 \\ V^2 \end{bmatrix} \quad (3)$$

$$\mathbf{I}^{abc} = \begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}; \mathbf{I}^{012} = \begin{bmatrix} I^0 \\ I^1 \\ I^2 \end{bmatrix} \quad (4)$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \end{bmatrix} \quad (5)$$

Based on (1) and (2), it can be shown that, in terms of phase voltage and current, the formulations of sequence

voltage and current are:

$$\mathbf{V}^{012} = \mathbf{A}^{-1} \mathbf{V}^{abc} \quad (6)$$

$$\mathbf{I}^{012} = \mathbf{A}^{-1} \mathbf{I}^{abc} \quad (7)$$

where:

$$\mathbf{A}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1\angle 120^\circ & 1\angle 240^\circ \\ 1 & 1\angle 240^\circ & 1\angle 120^\circ \end{bmatrix} \quad (8)$$

B. Impedance

The following is the derivation of phase and sequence impedance matrices for a three-phase electrical network. Since:

$$\mathbf{V}^{abc} = \mathbf{Z}^{abc} \mathbf{I}^{abc} \quad (9)$$

where:

$$\mathbf{Z}^{abc} = \begin{bmatrix} Z^{aa} & Z^{ab} & Z^{ac} \\ Z^{ba} & Z^{bb} & Z^{bc} \\ Z^{ca} & Z^{cb} & Z^{cc} \end{bmatrix} \quad (10)$$

Then, by multiplying both sides of (9) with \mathbf{A}^{-1} , (9) can be rewritten as:

$$\mathbf{A}^{-1} \mathbf{V}^{abc} = \mathbf{A}^{-1} \mathbf{Z}^{abc} \mathbf{I}^{abc} \quad (11)$$

On using (2) and (6) in (11) will result in:

$$\mathbf{V}^{012} = \mathbf{Z}^{012} \mathbf{I}^{012} \quad (12)$$

where \mathbf{Z}^{012} is the sequence impedance matrix and has the following formulation:

$$\mathbf{Z}^{012} = \mathbf{A}^{-1} \mathbf{Z}^{abc} \mathbf{A} \quad (13)$$

C. Power

For three-phase power system, complex powers in each phase are calculated using:

$$S^a = V^a I^{a*} \quad (14a)$$

$$S^b = V^b I^{b*} \quad (14b)$$

$$S^c = V^c I^{c*} \quad (14c)$$

Based on (1) and (2), the above complex power equations can be rewritten in terms of sequence voltage and current as:

$$S^a = (V^0 + V^1 + V^2)(I^0 + I^1 + I^2)^* \quad (15a)$$

$$S^b = (V^0 + a^2 V^1 + a V^2)(I^0 + a^2 I^1 + a I^2)^* \quad (15b)$$

$$S^c = (V^0 + a V^1 + a^2 V^2)(I^0 + a I^1 + a^2 I^2)^* \quad (15c)$$

where: $a = 1\angle 120^\circ$ is the complex operator.

In vector/matrix form, (15) is formulated as:

$$\mathbf{S}^{abc} = \text{diag}(\mathbf{A} \mathbf{V}^{012}) (\mathbf{A} \mathbf{I}^{012})^* \quad (16)$$

where: $\mathbf{S}^{abc} = [S^a \quad S^b \quad S^c]^T$.

III. THREE-PHASE DSLF FORMULATION

A. Sequence Admittance of Distribution Line

As explained previously, the unbalance in power system comes from two things: (i) line/network unbalance, and/or (ii) load unbalance. However, for power distribution systems, the load unbalance is more dominant than the line unbalance. Therefore, in this paper, the distribution line or network will be considered to be balanced. This means that the line is assumed to be transposed or has the same phase spacing so that the mutual impedances are identical (or: $Z_{ab} = Z_{bc} = Z_{ac} = Z_m$). Moreover, the self impedances are also assumed to be the same (or: $Z_{aa} = Z_{bb} = Z_{cc} = Z_s$). Therefore, based on (10), the phase impedance matrix becomes [12]:

$$\mathbf{Z}^{abc} = \begin{bmatrix} Z^s & Z^m & Z^m \\ Z^m & Z^s & Z^m \\ Z^m & Z^m & Z^s \end{bmatrix} \quad (17)$$

On using (13), it can be shown that the phase impedance matrix (17) when converted into its sequence impedance matrix will have the form:

$$\mathbf{Z}^{012} = \begin{bmatrix} Z^0 & 0 & 0 \\ 0 & Z^1 & 0 \\ 0 & 0 & Z^2 \end{bmatrix} \quad (18)$$

where:

$$Z^0 = Z^s + 2Z^m \quad (19a)$$

$$Z^1 = Z^2 = Z^s - Z^m \quad (19b)$$

Furthermore, the sequence admittance matrix that corresponds to the sequence impedance matrix (18) is:

$$\mathbf{Y}^{012} = (\mathbf{Z}^{012})^{-1} = \begin{bmatrix} Y^0 & 0 & 0 \\ 0 & Y^1 & 0 \\ 0 & 0 & Y^2 \end{bmatrix} \quad (20)$$

where:

$$Y^0 = \frac{I}{Z^s + 2Z^m} \quad (21a)$$

$$Y^1 = Y^2 = \frac{I}{Z^s - Z^m} \quad (21b)$$

B. Load Flow Problem Formulation

On using (12) and (20), the bus (nodal) voltage/current equation in terms of sequence components can be written as:

$$\mathbf{I}^{012} = \mathbf{Y}^{012} \mathbf{V}^{012} \quad (22)$$

Since \mathbf{Y}^{012} is a diagonal matrix, (22) can be decoupled into three separate equations as follows:

$$I^0 = Y^0 V^0 \quad (23a)$$

$$I^1 = Y^1 V^1 \quad (23b)$$

$$I^2 = Y^2 V^2 \quad (23c)$$

Based on (23), for distribution system with multiple buses, the nodal equations in terms of sequence components are:

$$\mathbf{I}^0 = \mathbf{Y}^0 \mathbf{V}^0 \quad (24a)$$

$$\mathbf{I}^1 = \mathbf{Y}^1 \mathbf{V}^1 \quad (24b)$$

$$\mathbf{I}^2 = \mathbf{Y}^2 \mathbf{V}^2 \quad (24c)$$

where:

$\mathbf{Y}^0, \mathbf{Y}^1, \mathbf{Y}^2$: zero, positive, and negative sequence bus admittance matrices

$\mathbf{V}^0, \mathbf{V}^1, \mathbf{V}^2$: zero, positive, and negative sequence bus voltage vectors

$\mathbf{I}^0, \mathbf{I}^1, \mathbf{I}^2$: zero, positive, and negative sequence bus current vectors

For distribution system with n buses, the above bus admittance matrices, bus voltage vectors, and bus current vectors will have the forms:

$$\mathbf{Y}^k = \begin{bmatrix} Y_{11}^k & Y_{12}^k & \cdots & Y_{1n}^k \\ Y_{21}^k & Y_{22}^k & \cdots & Y_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ Y_{n1}^k & Y_{n2}^k & \cdots & Y_{nn}^k \end{bmatrix} \quad (25a)$$

$$\mathbf{V}^k = \begin{bmatrix} V_1^k \\ V_2^k \\ \vdots \\ V_n^k \end{bmatrix}; \mathbf{I}^k = \begin{bmatrix} I_1^k \\ I_2^k \\ \vdots \\ I_n^k \end{bmatrix} \quad (25b)$$

where: k = 0, 1, 2.

On the other hand, based on (16), the complex power at each bus of electrical distribution system is formulated as:

$$\mathbf{S}_{Gi}^{abc} - \mathbf{S}_{Li}^{abc} = \text{diag}(\mathbf{A}\mathbf{V}_i^{012}) (\mathbf{A}\mathbf{I}_i^{012})^* \quad (26)$$

with $i = 1, 2, \dots, n$

where \mathbf{S}_{Gi} and \mathbf{S}_{Li} are power generation and load at i-th bus, respectively.

Rearranging (24) and (26) will results in:

$$\mathbf{I}^0 - \mathbf{Y}^0 \mathbf{V}^0 = \mathbf{0} \quad (27a)$$

$$\mathbf{I}^1 - \mathbf{Y}^1 \mathbf{V}^1 = \mathbf{0} \quad (27b)$$

$$\mathbf{I}^2 - \mathbf{Y}^2 \mathbf{V}^2 = \mathbf{0} \quad (27c)$$

$$\mathbf{S}_{Gi}^{abc} - \mathbf{S}_{Li}^{abc} - \text{diag}(\mathbf{A}\mathbf{V}_i^{012}) (\mathbf{A}\mathbf{I}_i^{012})^* = \mathbf{0} \quad (27d)$$

Set of equations (27) is the formulation of power flow problem for three-phase distribution system in the forms of sequence components. In power flow analysis, the set of equations (27) must be solved simultaneously. The results are then used to evaluate the steady state performance of the distribution system. In summary, all of the quantities in the power flow formulation, i.e. the quantities to be calculated (unknown quantities) and the specified quantities (known quantities), for each system bus are shown in Table I.

TABLE I: KNOWN AND UNKNOWN QUANTITIES		
Bus	Known Quantities	Unknown Quantities
	$\mathbf{Y}^0; \mathbf{Y}^1; \mathbf{Y}^2; \mathbf{S}_L^{abc}$	$I^0; I^1; I^2$
Substation	$\mathbf{V}^{012} = \begin{bmatrix} 0 \\ 1\angle 0^\circ \\ 0 \end{bmatrix}$	\mathbf{S}_G^{abc}
	$\mathbf{Y}^0; \mathbf{Y}^1; \mathbf{Y}^2; \mathbf{S}_L^{abc}$	$I^0; I^1; I^2$
Load	$\mathbf{S}_G^{abc} = [0 \ 0 \ 0]^T$	$V^0; V^1; V^2$

C. Solution Technique and Starting Values

Since sets of equations (27) are nonlinear, iterative techniques (such as Newton-Raphson method or trust-region method) have to be used to solve the equations [13], [14]. To initialize the iteration process, the following starting values for the unknown variables can be applied.

- Substation power:

$$\mathbf{S}_G^{abc(0)} = \sum \mathbf{S}_{Li}^{abc} \quad (\text{total load in the system})$$

- Load bus voltages:

$$\mathbf{V}_i^{012(0)} = \begin{bmatrix} 0 \\ 1\angle 0^\circ \\ 0 \end{bmatrix}$$

- Bus currents:

$$\mathbf{I}_i^{012(0)} = \begin{bmatrix} 0 \\ \sum_{j=1}^n Y_{ij}^I \\ 0 \end{bmatrix}$$

IV. CASE STUDY

A. Test System

In case study, 10-bus distribution system as shown in Fig. 1 will be used to validate the proposed method. This system is adopted from [6] and has a system voltage of 8.66 kV. Data for this test system are shown in Tables II and III. It should also be noted that all of the calculations are performed on a personal computer (PC) with the help of MATLAB™ software.

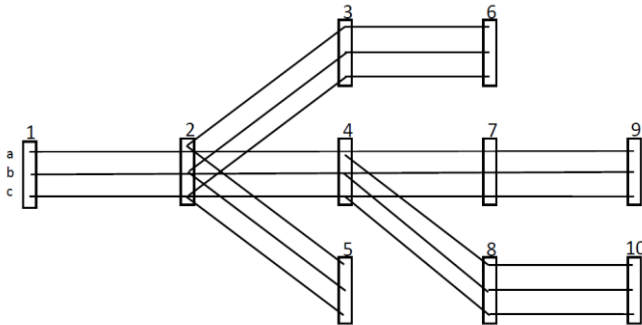


Fig. 1. 10-bus distribution test system.

TABLE II: 10-BUS SYSTEM LINE IMPEDANCE (OHM)

No.	Line	Self Impedance	Mutual Impedance
1	1-2	1 + j2	j0.5
2	2-3	1 + j	j0.25
3	2-4	1 + j2	j0.5
4	2-5	1 + j2	j0.5
5	3-6	1 + j2	j0.5
6	4-7	1 + j	j0.25
7	4-8	1 + j	j0.25
8	7-9	1 + j	j0.25
9	8-10	1 + j2	j0.5

TABLE III: 10-BUS SYSTEM LOAD (KW, KVAR)

Bus	Phase a		Phase b		Phase c	
	P	Q	P	Q	P	Q
1	0	0	0	0	0	0
2	50	25	50	25	50	12.5
3	50	25	37.5	12.5	37.5	25
4	50	25	25	25	25	25
5	37.5	25	37.5	12.5	50	12.5
6	100	75	50	75	100	50
7	100	75	100	12.5	25	12.5
8	100	75	25	12.5	50	25
9	10	12.5	10	25	25	12.5
10	10	25	25	12.5	10	12.5

B. Results and Discussion

Results of the power flow study in terms of bus voltages are shown in Table IV. For comparison purposes, results from the phase components based power flow method are

also shown in Table V. It can be seen that the results are in exact agreement which confirms the validity of the proposed method.

TABLE IV: VOLTAGE PROFILE OF 10-BUS DISTRIBUTION NETWORK (PROPOSED METHOD)

Bus	V _a	V _b	V _c
1	1.00000∠0.00°	1.00000∠-120.00°	1.00000∠120.00°
2	0.95072∠-1.13°	0.97586∠-120.86°	0.97205∠119.37°
3	0.94132∠-1.17°	0.96967∠-120.79°	0.96357∠119.30°
4	0.92114∠-1.79°	0.96670∠-120.44°	0.96070∠119.36°
5	0.94750∠-1.20°	0.97347∠-120.97°	0.96934∠119.22°
6	0.93278∠-1.38°	0.96274∠-120.76°	0.95584∠119.03°
7	0.91226∠-1.79°	0.96175∠-120.61°	0.95822∠119.39°
8	0.91234∠-1.78°	0.96482∠-120.49°	0.95674∠119.39°
9	0.91160∠-1.76°	0.96029∠-120.56°	0.95685∠119.36°
10	0.90970∠-1.75°	0.96315∠-120.58°	0.95595∠119.41°

TABLE V: VOLTAGE PROFILE OF 10-BUS DISTRIBUTION NETWORK (PHASE COMPONENTS METHOD)

Bus	V _a	V _b	V _c
1	1.00000∠0.00°	1.00000∠-120.00°	1.00000∠120.00°
2	0.95072∠-1.13°	0.97586∠-120.86°	0.97205∠119.37°
3	0.94132∠-1.17°	0.96967∠-120.79°	0.96357∠119.30°
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V. CONCLUSION

Power flow is an important tool in operational study and planning of electric power systems. The single-phase power flow algorithm assumes that the unbalance in an electric power system is small and can be neglected so that the system can be considered to be balanced. There are many cases where system unbalance can no longer be ignored, and therefore, three-phase power flow algorithm has to be used in the analysis. A new technique for three-phase distribution system power flow analysis using sequence components has been proposed and developed in this paper. The application of sequence components has the advantage that the size of the problem can effectively be reduced, and solution to the power flow problem will be easier to find. Comparative studies between the proposed method and the phase components based method have also been carried out to validate the proposed power flow method.

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